

# Analysis of agents speculation on settlement results for different types of pricing mechanisms

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**Abstract**—In the paper we present, discuss and compare various methods of constraints costs allocation on infrastructure market systems, using the intra-day power market and network capacity constraints as an example. We briefly describe few pricing methods arising from different approaches and compare their features experimentally. The main aim of presented experiments is to examine incentives for market players generated by discussed methods. We examine if market players have motivations to reveal their true marginal costs and if trade profits are appropriately distributed among all market players and system operator.

## I. INTRODUCTION

System constraints play an essential role in many infrastructure market systems. They limit possibilities of commodities exchange and reduce trade profits as well as disturb conditions of fairly market entities treatment. System constraints may also allow some entities to gain market power. Therefore appropriate pricing and constraints costs allocation methods are very important for the proper market performance.

In the paper we focus on inspecting incentive compatibility for the set of pricing mechanisms on the intra-day electric power markets. We investigate how the pricing mechanisms impacts on the producers to reveal their true costs. In the case studies we analyze results of every pricing mechanism against the background of producers rising their offer prices. The purpose of experiments is to examine incentive compatibility properties of the presented pricing mechanisms. The aim of this paper is to investigate behaviour of some number of pricing mechanisms in the case of producers which are trying to maximize their profits by rising their offer prices.

## II. THE “COPPER-PLATE” POWER MARKET MODEL

On the Polish electrical energy market as well as on many other European markets the “copper-plate” market model is used. In these model energy is balanced in an aggregated way. There is only one energy balance equation and one price for energy in the whole system. Each offer on the market may be next saddled with some part of constraints costs, so final prices for particular market entities may be diverse.

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For the purposes of this paper we consider simple copper-plate intra-day balancing market model with limited network capacity constraints. A general formulation of a (single-period) quantitative balancing model can be stated as follows:

$$\tilde{Q} = \max_{d,p} [Q = \sum_m e_m d_m - \sum_j c_j p_j] \quad (1)$$

$$\sum_m d_m - \sum_j p_j = 0 \quad (2)$$

$$\sum_v w_{ve} (\sum_{j \in J_v} p_j - \sum_{m \in M_v} d_m) \leq Q_e \quad \forall e \quad (3)$$

$$0 \leq p_j \leq p_j^{max} \quad \forall j \quad (4)$$

$$0 \leq d_m \leq d_m^{max} \quad \forall m \quad (5)$$

Variables  $p_j$  and  $d_m$  represent accepted volume for sell and buy offers respectively,  $c_j$  and  $e_m$  are offer prices,  $p_j^{max}$  and  $d_m^{max}$  are maximal offered volumes, and  $Q_e$  is capacity of power line  $e$ . Constraint (2) is the “copper-plate” energy balance for the system. The objective function maximizes the market trade surplus. Network capacity constraints (3) are modeled by using the power distribution factors (PTDF) [4], denoted as  $w_{ve}$  for node  $v$  and edge  $e$ .  $J_v$  and  $M_v$  are the sets of offers submitted in given node  $v$  by sellers and buyers respectively. The model can be easily extended to the multi-period case with respecting other system constraints, as it was shown e.g. in [6].

## III. POWER MARKETS PRICING METHODS

Presence of system constraints (3) in market balancing models causes the trade profits decrease, which may be considered as the system constraints cost. The total costs of system constraints can be defined as  $c(N) = \tilde{Q}(\emptyset) - \tilde{Q}(N)$ , where  $\tilde{Q}(\emptyset)$  is the trade profit which can be achieved in case of constraints absence (all lines have infinite capacity), whereas  $\tilde{Q}(N)$  is the reduced profit achieved when respecting the whole set  $N$  of system constraints. In this section we present various methods of allocation cost  $c(N)$  to individual constraints or offers. Let  $c(S)$  denote the joint costs of system constraints when only subset  $S$  of constraints is considered and rest are omitted. Generally, constraints’ costs are not subadditive, so for some subsets of constraints  $S1, S2$  usually  $c(S1) + c(S2) \neq c(S1 \cup S2)$ , that is sum of constraints contained in  $S1$  cost and constraints contained in  $S2$  cost is different from cost caused by all these constraints considered together. Therefore, problem of constraints’ costs

allocation is nontrivial. The cost allocation methods presented in the paper are based on diverse methodologies: simple heuristics, dual variables, game theory approach, and parametric analysis.

#### A. Two-state comparison

On the Polish electrical energy market the System operator (TSO) uses a rough approach. Two states of constraints set are compared – results of constrained balancing and unconstrained case. On that basis TSO decides who is responsible for cost of constraints. If schedule of a unit is different in the constrained plan compared to the unconstrained plan then this change is considered as forced by constraints and units transaction prices may be cut down. Force changes cannot create unit’s local market power. This mechanism does not cover the costs but tries to mitigate local market power. Costs of constraints are balanced in the long term horizon by surplus on the market held by the TSO in an arbitrary way and network tariffs set apriori and covered mainly by energy consumers.

#### B. Locational marginal pricing

Locational marginal pricing (LMP) is widely applied method for the network constraints consideration especially related to congestions and transmission losses. The concept of LMP was introduced by Schweppe et al. [5] and extended by Hogan [2]. The LMP method allows one to calculate marginal prices of feeding (or withdrawing) energy in individual nodes of the infrastructure network. Price diversification between two adjacent nodes occurs when the transmission capacity between the two nodes is limited and exhausted, or losses appear. To formulate the LMP model, we have to replace balance equation (2) with the set of balance equations for all network nodes (7) and add some constraints representing physical laws concerning power flows (8) - (10).

$$\max_{d,p} \left[ \sum_m e_m d_m - \sum_j c_j p_j \right] \quad (6)$$

$$\sum_{w=1}^N P_{vw} - \sum_{j \in J_v} p_j + \sum_{m \in m_v} d_m = 0 \quad \forall v \quad (7)$$

$$P_{vw} = V_v V_w Y_{vw} (\theta_v - \theta_w) \quad \forall (v, w) \in E \quad (8)$$

$$\theta_0 = 0 \quad (9)$$

$$-Q_{vw} \leq P_{vw} \leq Q_{vw} \quad \forall (v, w) \in E \quad (10)$$

LMP provides direct cost allocation to specific offers by setting transaction prices for these offers. Dual variables -  $\lambda_v$  related to the energy balance equation for each network node (7) are used as the energy prices for appropriate nodes. Cost of network constraints for given line is represented as a difference between energy prices in adjacent nodes.

#### C. SCRБ

The Separable Costs Remaining Benefits (abbr. SCRБ) method is an arbitrary method arising from civil engineering problems of allocating costs of multipurpose reservoirs [8]. Cost  $x_i$  allocated to constraint  $i$  is divided into *separable cost* -  $s_i$  and *remaining cost* -  $R$ . Separable cost of constraint is equal to cost of its inclusion to all other constraints  $s_i = c(N) - c(N \setminus i)$ . Remaining cost:  $R = c(N) - \sum_{i \in N} s_i$  is allocated proportionally to remaining benefits  $r_i = c(i) - s_i$ , that is cost of respecting solely constraint  $i$  minus so far allocated separable costs, thus finally cost  $x_i$  allocated to the constraint  $i$  may be determined as:

$$x_i = s_i + \frac{r_i}{\sum_{j \in N} r_j} R \quad (11)$$

#### D. Shapley value

This method derives the approach from game theory. In our game each capacity constraint is considered as a player. The Shapley value represents the average player contribution into constraints’ cost for all orders of players joining into the game [8]. To compute Shapley value, the following equation can be used:

$$x_i = \sum_{S \subseteq N, i \in S} \frac{|S \setminus i|! |N \setminus S|!}{|N|!} [c(S) - c(S \setminus i)] \quad (12)$$

For each  $S \subseteq N$  (each subset  $S$  of constraints set  $N$ ) the joint cost  $c(S)$  has to be calculated. Thus this method has much bigger complexity than SCRБ, but it explores more possible dependencies between constraints.

#### E. Aumann-Shapley pricing

Aumann-Shapley pricing address the complexity problem arising with Shapley value [8]. Let vector  $\mathbf{b}$  be values of right hand side of all network constraints (capacities) and  $F(\mathbf{b})$  be constraints cost for the values  $\mathbf{b}$ . Aumann-Shapley price for player  $i$  is an average marginal cost of constraint  $i$  along the ray from  $\mathbf{b}^0$  (all constraints completely relaxed) to the target  $\mathbf{b}^*$  (nominal values of right hand sides):

$$x_i = \int_0^1 [\partial F(t(\mathbf{b}^* - \mathbf{b}^0) + \mathbf{b}^0) / \partial b_i] dt \quad (13)$$

As distinct from Shapley value and SCRБ some fractional coalitions are taken into consideration, not only the cases of infinity or nominal capacities.

#### F. MASIT

In this approach an allocation which is “minimal” in some meaning and satisfies the incremental cost test (15) is searched. This condition states that the cost allocated to all constraints in subset  $S$  is not lower than the cost caused by introduction this constraint into the market. This condition has to be met for all subsets of constraints  $S \subseteq N$ . We may find this allocation by solving some auxiliary LP problem:

$$\min_{i \in N} x_i \quad (14)$$

$$\sum_{i \in S} x_i \geq c(N) - c(N \setminus S) \quad \forall S \subseteq N \quad (15)$$

$$x_i \geq 0 \quad \forall i \in N \quad (16)$$

Above problem is multi-criteria and has nice properties if equitable preferences model is used to find the solution of (14)-(16) [3]. In this approach the total cost allocated to all constraints may be higher than  $c(N)$ .

#### IV. COSTS ALLOCATION TO OFFERS

Except LMP and “Two-state comparison” approaches, which determine directly the market prices, all remaining methods allocate system costs to specific constraints. To clear the market we have to allocate constraints’ costs to specific offers and determine market transaction prices -  $\pi_j$  for each offer. Price for a sell offer in node  $j$  can be calculated as:  $\pi_j = \pi_0 - \pi_{j0}$ , and for a buy offer in node  $j$  as:  $\pi_j = \pi_0 + \pi_{0j}$ , where  $\pi_0$  is energy price in the “copper-plate” virtual node and  $\pi_{j0}$  and  $\pi_{0j}$  are prices for power flow from node  $j$  to “copper-plate” and from “copper-plate” to node  $j$  respectively, called *node fees*. We have to solve two problems: determine “copper-plate” energy price -  $\pi_0$  and prices  $\pi_{j0}/\pi_{0j}$ , which derived from constraints costs. This approach was described by Toczyłowski in [7] and is depicted in Fig. 1. In fact  $\pi_{j0}$  and  $\pi_{0j}$  may be negative, which increase prices for sell offers and decrease for buy offers.

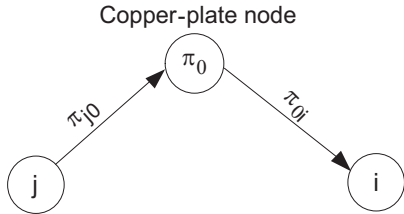


Fig. 1. Prices for power flow to and from the copper-plate node

As the “copper-plate” market energy price  $\pi_0$  we use marginal energy price on the balancing market. Prices  $\pi_{j0} / \pi_{0j}$  are derived from the constraints costs -  $x_i$ , which has been determined earlier. First we have to determine the influence of the load or generation in node  $j$  on flow in line  $e$ . To do this we use tracking method proposed by Bialek in [1] which assumes that inflows are shared proportionally between outflows - the network node is a perfect “mixer”. After determining the influence of net load/generation in node on the flow in each line we distribute it among all loads and generators in this node proportionally to their production or consumption. Then we allocate cost of each network constraint  $x_i$  to specific offers according to calculated shares in each flow as the *node charges* -  $k_j$ .

If the system operator charges market entities by the nodal charges he gets some surplus in amount  $c(N)$  using SCRB/Shapley/Aumann-Shapely mechanisms or higher in case of using LMP or MASIT mechanisms. If we do not consider network losses operator should not turn a profit on network constraints, so this surplus should be redistributed among market entities. Therefore market entities which suffer a lose due to network constraints should get

form operator’s surplus some compensation in amount:  $r_j = (\pi_0 - c_j)(p_{0j} - p_j)$ , where  $\pi_0$  is market price on unconstrained market,  $c_j$  is unit  $j$  offer price,  $p_{0j}$  is volume accepted on unconstrained market and  $p_j$  is volume accepted on market with constraints. Finally payment for entity  $j$  can be determined as:

$$w_j = \pi_0 p_j + r_j - k_j \quad (17)$$

#### V. INCENTIVES ANALYSIS

Participants of the intra-day markets may speculate by changing their offer prices. In the paper we consider approach which is based on simple case study, in which we try to rise the offer price for one of producers. Along growing offer price the results of pricing mechanisms changes and we can draw some conclusions on the basis of obtained results.

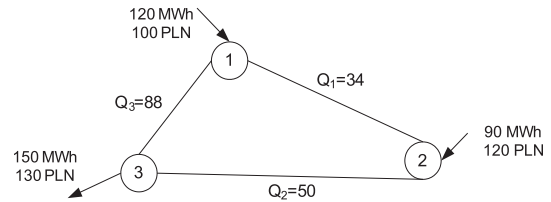


Fig. 2. Considerd network

We prepared framework system for testing and comparison various approaches to constraints and market pricing and used it for experiments. At the beginning we consider simple example with three nodes presented on Fig. 2. Let us assume existence of two producers, located in first and second node and one consumer located in third node. Every market participant submits one elastic offer, values of particular offers are presented on figure. Network has three edges, every one is constrained in the respect of maximum acceptable flow.

#### VI. PARAMETRIC ANALYSIS

The SCRB and SHAPLEY mechanisms brings identical results (which come from the test framework configuration), so we present the results of this mechanisms together. The MASIT mechanism brings almost the same results, in a sense of incentive motivation, as a mechanisms SCRB/SHAPLEY, so we disregard the results in the paper.

On the Fig. 3, 4 we can observe results of parametric analysis in which first producer tries to reach bigger profit by changing his offer price, the settlement is done by LMP mechanism.

We can notice, that for LMP mechanism, every offer results in exactly the same price, it follows that first producer has the market power, so he has incentives to bid untruthfully.

The results of SCRB/SHAPLEY mechanisms are presented on the Fig. 5, 6. Until the first producer do not submit higher offer than the second producer, that is until he do not become a marginal player, his price and income are rising slightly with decreasing offer price, so he’s got a little incentives for submitting untruthful offers. This effect is caused by increasing compensation for the producer -



Fig. 3. Results of parametric analysis for the first producer – income, LMP mechanism



Fig. 4. Results of parametric analysis for the first producer – prices, LMP mechanism

when the difference between offer price and price on the market without constraints rising, the market participants accepted volume is reduced by the network constraints, the compensation is rising either. On the other hand, participant which is trying to speculate by lowering his offer price risks, that he could be accepted on lower price than his real production price. When producer's offer exceed the offer of the second producer and he become a marginal player, his price and income begins to increase along increasing offer price. However, the pricing mechanisms are trying to punish the participant for speculation, as we can see on the Fig. 5, 6, the node charge is rising when the participant is trying to speculate. Moreover participant risks that his offer will be rejected if he rise it too high.

Similar experiment was realized for the second producer. On Fig. 7, 8 we can observe results for LMP mechanism.

As the second producer is a marginal player not only for mechanism based on LMP but for all other mechanisms, rising offer price brings in result increase of price in second node and rise of producers income. After exceeding a recipients offer price by producers offer price, the selling price in second node becomes greater than buying price in third node. This is very disadvantageous feature, because producer

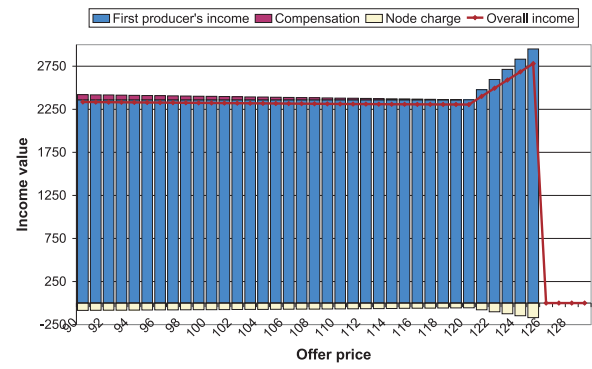


Fig. 5. Results of parametric analysis for the first producer – income, SCRB/SHAPLEY mechanism

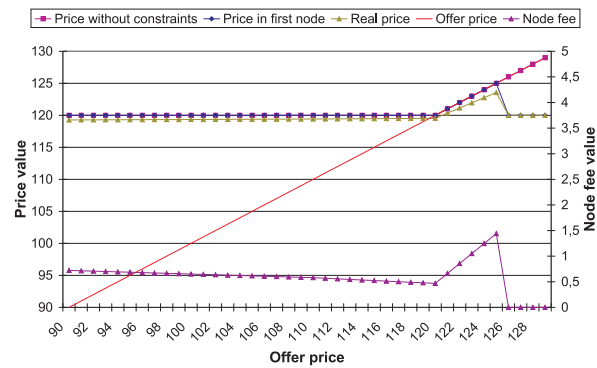


Fig. 6. Results of parametric analysis for the first producer – prices, SCRB/SHAPLEY mechanism

could derive undeserved profits from this situation, e.g. make a coalition with recipient in third node.

Let us focus our attention on Fig. 9, 10 where analysis results of SCRB/SHAPLEY mechanism are depicted. The disadvantageous feature, namely the situation where the selling price in one node is greater than buying price in another node does not occur in this mechanisms. Producer, as he is a marginal player, has incentives to rising his offer price above his truthful offer price. However he has got as well incentives to lower his offer prices, because his work point is hardly restricted by a network constraints, and the bigger difference between his offer price and the price from market without constraints, the bigger the compensation is, he tries to become non-marginal player and lower as much as possible his offer price. The mechanisms tries to restrict the incentives for this speculation by increasing the node fee, but it could not dispose it absolutely. In the SCRB/SHAPLEY mechanisms, occurs another disadvantage, namely when producer submits truthful bid, he obtain negative income, so these mechanisms encourage the marginal player for bidding untruthfully.

It is necessary to observe, that in conditions of real trade, when the second producer is trying to obtain some

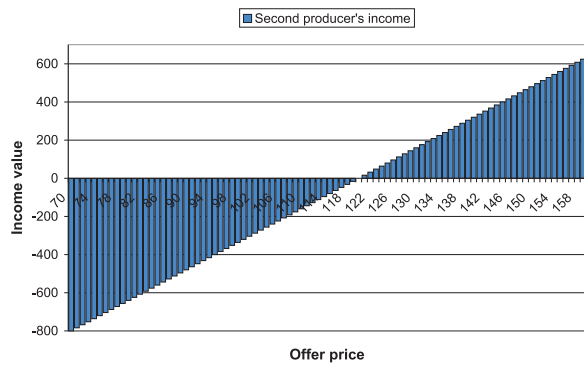


Fig. 7. Results of parametric analysis for the second producer – income, LMP mechanism

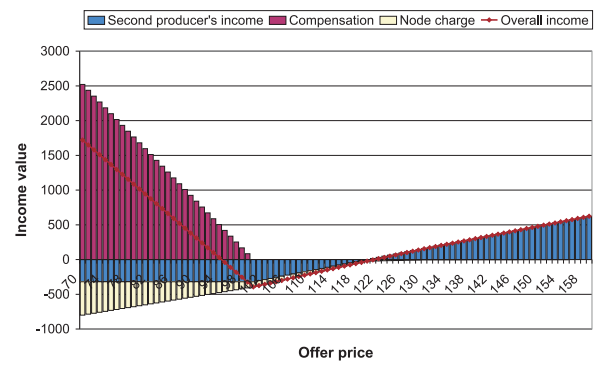


Fig. 9. Results of parametric analysis for the second producer – income, SCRB/SHAPLEY mechanisms

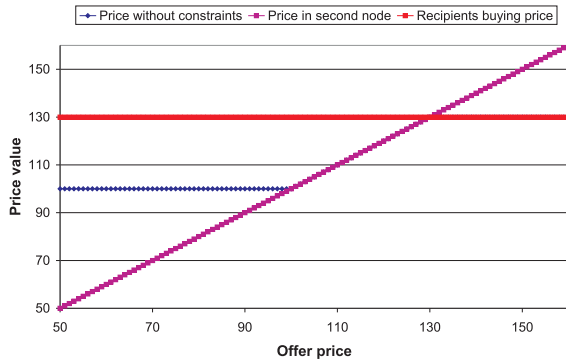


Fig. 8. Results of parametric analysis for the second producer – prices, LMP mechanism

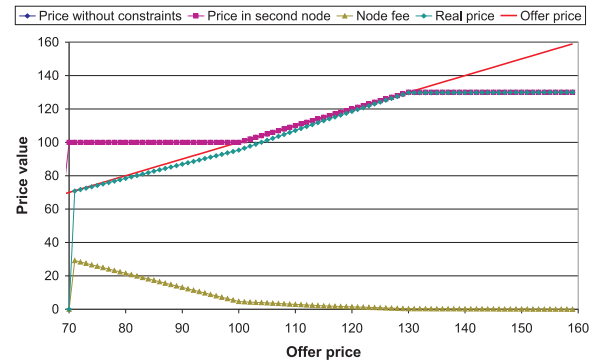


Fig. 10. Results of parametric analysis for the second producer – prices, SCRB/SHAPLEY mechanisms

positive profit by lowering his offer price, and making use from compensation mechanism, the first producer incur loses (see Fig. 11). Thus, if the second producer will be lowering his offer price, and gaining some positive profit, and simultaneously, the first producer will be incurring loses, the first producer can quit the game. In this situation, the second producer will become a marginal player, and will sell the energy for his offer price, which is much smaller than his production price. In case when the second producer is trying to reach bigger income by increasing his offer price, consumers income is lowering (see Fig. 11). When the second producer offer price crosses the consumers offer price, the consumers income becomes negative. Analogously to previous situation the consumer can quit the game by reason of negative income. In this situation the second producer, despite of its income increase, can speculate only in small range: from its production price to the consumers offer price. Thus, described situation shows us that advantages from speculation depends strongly on the behaviour of others participants of the game.

On the Fig. 12 we can observe the operator's surplus for all inspected mechanisms. We can notice, that for

SCRB/SHAPLEY mechanisms, the operator's surplus is always equal zero, which is positive feature, by reason of the operator should not earn any profits from settlement. In MASIT mechanism, operator has small positive profit, which is very small in comparison to surplus obtained by operator in LMP mechanism. Very large operator's surplus in the LMP mechanism is a big fault, because the operator should not earn any money from settlement.

## VII. SUMMARY

In the paper we analyze some number of pricing mechanisms in respect of incentive compatibility and speculation encouragements. One of studied methods, namely Location Marginal Pricing, is widely applied in many power systems. This forces necessity that it should be a model mechanism which can be used as reference to other, newly forming pricing methods. Unfortunately, case study supplied us with some undesirable outcomes.

In each of studied pricing mechanism, marginal player has incentives to submit untruthful offers. In LMP mechanism this is emphasized, because in every node exists a marginal player. This mechanism has also more unfavourable feature, that is a selling price in one node could be, with particular



Fig. 11. Income value for market participants, the second producer changes his offer price – SCR/SHAPLEY mechanisms

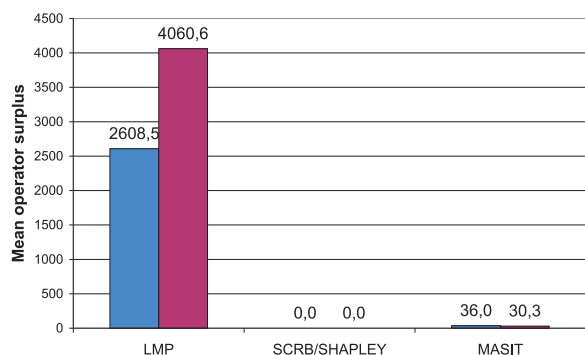


Fig. 12. Mean operator's surplus for inspected mechanisms, particular graphs concerns situation when first or second producer are trying to speculate

combination of flow constraints, greater than buying price in other node. This feature is very dangerous for whole market, because some of market entities could conclude an coalition and derive some undeserve profits. On the other hand, mechanisms SCR, SHAPLEY and MASIT have incentives to submitting untruthful offers, especially for participants who are hardly restricted by a network constraints, namely to lower their offers. The reason for submitting lower offer prices is to obtain a compensation. The consecutive negative feature of these mechanisms is fact, that when a marginal producer submits a truthful offer, compatible with its production cost, he obtains small negative profit. This is an incentive to submitting a little bigger offer prices, introduced by mechanism. Therefore if the mechanism forces us to submit a little bigger offer, why do not rise the offer price more? This feature is very negative, because the mechanism introduces an incentive for bidding untruthfully. Another disadvantage of LMP mechanism is fact that the operator gain enormous surplus which should be given to competitive market participants. In the SCR and SHAPLEY mechanisms, operator always gain zero profit,

which is positive feature. In the MASIT mechanism operator gain very small surplus, which came from the network cost division fact.

All inspected pricing methods, are not devoid of negative features, however broadly used mechanism should be. Obviously, utilized framework system for testing is very simple, but existence of some negative features on such a simple system is unacceptable.

## REFERENCES

- [1] Bialek J.: Tracing the flow of electricity, *IEE Proc.-Gen. Transm. Distrib.* 143, 313-320, 1996.
- [2] Hogan W. W.: Contract networks for electricity power transmission, *Journal of Regulatory Economics*, vol. 4(3), pp. 211-242, Nov. 1992.
- [3] Kaleta M.: A method for infrastructure costs allocation free from subsidies during market balancing, *Zeszyty Naukowe Politechniki Śląskiej*, z. 145. pp. 105-112, 2006 (in Polish).
- [4] Sauer P.W.: On the formulation of power distribution factors for linear load flow methods. *IEEE, Transactions on Power Systems*, Vol. 100, pp. 1001-1005, 1981.
- [5] Schwepe F.C.: *Spot pricing of electricity*, New Kluwer Academic Publishers, NY, 1988.
- [6] Smolira K., Toczyłowski E.: Real-time Market Mechanisms for Control in Distributed Networks, Proc. 12th IEEE International Conference on Methods and Models in Automation and Robotics, 2006.
- [7] Toczyłowski E.: *Optimization of Market Processes under Constraints*, EXIT Academic Publishing, Warsaw, 2002 (in Polish).
- [8] Young H. P., *Cost Allocation: Methods, Principles, Applications*, North-Holland, New York, 1985.