

Model for Balancing Aggregated Communication Bandwidth Resources

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Abstract

We present a multicommodity bandwidth exchange model BACBR – *Balancing Aggregated Communication Bandwidth Resources* – that allows us to aggregate similar offers. In this model offers submitted to sell (or buy) the same, similar, or equivalent network resources (or demands for end-to-end connections) are aggregated into single commodities. BACBR model is based on the BCBT model [13]. It requires much less variables and constraints than original BCBT, however the outcomes need to be disaggregated. The general model for disaggregation is also given in the paper.

bandwidth market, auctions, market clearing, multi-commodity trade, aggregation

1 Introduction

The multicommodity exchange models are promising tools that allow to answer emerging requirements for efficient, optimized trading mechanisms well suited on the competitive bandwidth markets. A complex and dynamic bandwidth trading environment is broadly believed to be developed [2, 5, 12, 11], as new technological and conceptual opportunities are rapidly appearing. For new markets, requiring thousands of bids and offers to be auctioned, the today most popular communication bandwidth trading tools, such as bilateral agreements, or current simple auctions and exchanges (that aim mainly in facilitating buyer-seller contacts), are not sufficient.

For the purpose of modeling trade of bandwidth resources in the communication networks, we assume that the network consists of nodes connected by links. The inter-node link may represent a net-

work resource, that can be an elementary commodity on the bandwidth market. However, network resources being traded can be more complex and can be composed of many parallel links, or end-to-end node connections represented by paths, sub-networks.

In this paper we state a multicommodity bandwidth exchange model – Balancing Aggregated Communication Bandwidth Resources (BACBR), that considers aggregation of offers submitted to buy or sell the same, similar, or equivalent commodities related to the network resources. The bandwidth trading is considered from the viewpoint of many network operators, service providers and other wholesale active market players, buying and selling bandwidth.

We believe that the current research proposals for auctioning bandwidth are still insufficient to address diverse market participants needs and requirements. One such a need is the end-to-end network paths trading under competition – when multiple parallel link resources can be offered for sale, or multiple end-to-end connections are bidding.

To cope with the problem of providing bidders with possibility of submitting offers for bundles of elementary commodities when auctioning bandwidth, researchers have proposed two approaches: (i) simultaneous, single link auctions [1, 3, 4, 10]; and (ii) combinatorial auctions [8]. In the first approach special, iterative mechanisms are required to coordinate individual links-auctions. The second approach requires buyers to specify the particular links that constitute a desired path. Both approaches lead to welfare inefficiency, as was shown in [13].

The auction BACBR model stated in this paper falls into a class of the multicommodity exchange models, that provide efficient resources allocation

solving global economic welfare maximization problem [6, 7, 9, 13]. Multicommodity means that market entities (further called bidders) can trade with bundles (packages) of different commodities. The Balancing Communication Bandwidth Trade (BCBT) model proposed in [13] allows bidders to place buy offers not for bundled links, but rather for end-to-end connections. Therefore buyer does not have to know which links to choose to best allocate the demanded capacity. It is the decision model that allocates the most efficient links to paths.

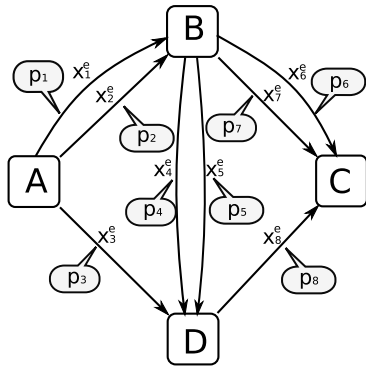


Figure 1: Resource graph for network links modeled as the multigraph. One offer concerns one particular network link

Basic BCBT model interprets individual bandwidth buy and sell offers as separate elementary commodities that correspond to traffic demand and network links respectively. Then, in case of many market participants offering bandwidth on links connecting the same network nodes or demanding the same connections paths, the trade is performed upon a multigraph – see Fig. 1 and Fig. 2. It means that in the case of BCBT model, only one offer is submitted on the particular commodity.

In real world, with possibly thousands of bids being auctioned, such an approach may become inefficient. Thus an aggregation of the BCBT model is required to reduce the complexity.

The aggregated BACBR model considers aggregate commodities structure modeled as a simple graph – see Fig. 3, Fig. 4. It means that in the BACBR model multiple offers can be submitted on aggregate link or end-to-end connection, considered as an aggregate commodity.

The rest of the paper is organized as follows. The mathematical statements both of BCBT and new

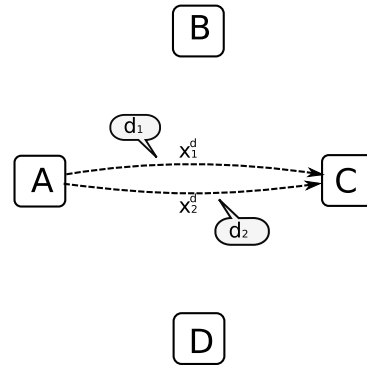


Figure 2: Resource graph for network demands modeled as a multigraph. One offer concerns particular network demand

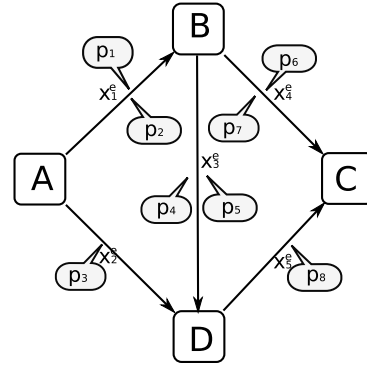


Figure 3: Resource graph for network demands modeled as the simple graph. Multiple offers for the particular network resources exist

BACBR models are given in next section. We also show, that the aggregated model BACBR has all positive features of the simple BCBT model, such as maximization of global economic surplus and possibility of placing buy offers not for bundled links, but for end-to-end connections. Moreover, the BACBR requires much less variables and constraints than original BCBT. What is also important, the market prices for aggregate commodities can be determined on competitive grounds. As the detailed realization of particular offers is not given in the solution of BACBR model so the disaggregation is needed. The process of disaggregation may also have some advantages as it can consider various individual constraints and requirements. The analysis of disaggregation techniques as well as gen-

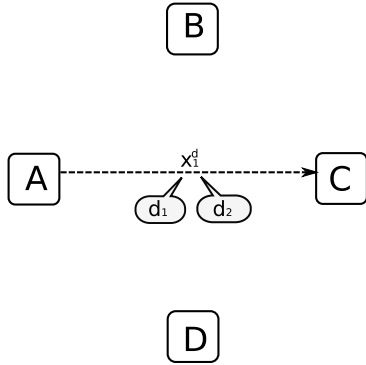


Figure 4: Resource graph for network demands modeled as the strict graph. Multiple offers for particular network resources exists

eral model for disaggregation is given in section 3. In section 4 we summarize our findings.

2 Mathematical model

We assume that the communication network consists of nodes connected by links. The inter-node link may represent a network resource (bandwidth), that can be an elementary commodity offered for sale on the bandwidth market. However, network resources being traded can be more complex and can be composed of many parallel links, or end-to-end node connections represented by paths or subnetworks.

Every buy offer concerns a point-to-point bandwidth connection between a pair of specified locations in a communication network. The locations form a set of network nodes V . The connections (and links) are unidirectional, i.e. they have source and sink nodes. First we briefly report the conventional model BCBT to allow us for the extension discussion.

2.1 The BCBT model

The objective of BCBT model [7], [13] is the maximization total economic welfare (1), which is the sum of total buyers and sellers surpluses. The constraints (2) and (3) set upper and lower bounds on particular network links (x_e) and particular end-to-end network demands (x_d). The non-negative variable x_{ed} (5) is interpreted as a bandwidth capacity

allocated to network link e to serve end-to-end demand d . Also, the sum of capacities allocated to all network demands $\sum_{d \in D} x_{ed}$ served by particular network link e , should not exceed the realization x_e of the link (4). Finally, the sum of all capacities, provided with incidence matrix a_{ve} , allocated to all network links, serving particular network demand, should not exceed the realization of the end-to-end demand x_d (6).

$$\hat{Q} = \max \left(\sum_{d \in D} E_d x_d - \sum_{e \in E} S_e x_e \right) \quad (1)$$

$$0 \leq x_d \leq h_d, \quad \forall d \in D \quad (2)$$

$$0 \leq x_e \leq y_e, \quad \forall e \in E \quad (3)$$

$$\sum_{d \in D} x_{ed} \leq x_e, \quad \forall e \in E \quad (4)$$

$$0 \leq x_{ed}, \quad \forall e \in E, d \in D \quad (5)$$

$$\sum_{e \in E} a_{ve} x_{ed} = \begin{cases} x_d & v = s_d \\ 0 & v \neq s_d, t_d \\ -x_d & v = t_d \end{cases}, \quad \forall v \in V, d \in D \quad (6)$$

where: **indices:**

$d = 1, 2, \dots, D$ buy offers – demands for bandwidth

$v = 1, 2, \dots, V$ network nodes

$e = 1, 2, \dots, E$ sell offers – network resources

parameters:

$a_{ve} = 1$ if link e originates in node v ,

$= -1$ if e terminates in node v ,

$= 0$ otherwise

s_d source node for demand d

t_d sink node for demand d

h_d required capacity of demand d

E_d offered unit price for demand d

y_e offered capacity of network link e

S_e offered unit price for network link e

variables:

x_{ed} bandwidth flow serving demand d allocated to network link e

x_d contracted bandwidth capacity for demand d

x_e contracted bandwidth capacity for network link e

The x_e and x_d are respectively values of realized bandwidth on the link e and the demand d . They are also the accepted offers for link e and demand d – in the BCBT model sell offers correspond network links and buy offers correspond demand paths resulting in a multigraph. It means, that for a single commodity only one offer can be

submitted. However, in case of competitive market with many participants demanding the same connections paths or offering bandwidth on links connecting the same network nodes, the size of the resource graph would be enormous.

2.2 BACBR model

Now we propose a nontrivial extension to BCBT model – the model for Balancing Aggregated Communication Bandwidth Resources (abbrev. BACBR), where multiple offers for selling a single network aggregate resource (i.e. network link), or for buying the same connections, are handled in an aggregate manner.

Let us replace the x_e variable by the sum of all adequate bandwidth realizations p_l of sell offers concerning particular network link e :

$$x_e = \sum_{l \in S(e)} p_l, \quad \forall e \in E \quad (7)$$

Variable p_l is a realization of l -th offer for selling link ($e : l \in S(e), S(e') \cap S(e'') = \emptyset, \forall e', e'' \in E, e' \neq e''$), $S(e) \subset S$ is a subset of sell offers S concerning particular link e . Thus, we obtain the aggregation of all sell offers submitted on specified link e ($\sum_{l \in S(e)} p_l$).

Analogously, let us replace the x_d variable by the sum of all adequate bandwidth realizations d_m of buy offers concerning particular end-to-end connection d :

$$x_d = \sum_{m \in B(d)} d_m, \quad \forall d \in D \quad (8)$$

Variable d_m is a realization of m -th offer for buying demand ($d : m \in B(d), B(d') \cap B(d'') = \emptyset, \forall d', d'' \in D, d' \neq d''$), $B(d) \subset B$ is a subset of buy offers B concerning particular demand d . Thus, we obtain the aggregation of all buy offers submitted on specified connection d ($\sum_{m \in B(d)} d_m$).

Last, we need to change the notation of offer parameters: the offer price we denote as s_l for l -th sell offer, and e_m for m -th buy offer. The maximal volume of bandwidth, associated with the l -th offer we denote as p_l^{max} , analogously, the maximal volume of bandwidth, associated to m -th offer we denote as d_m^{max} .

Finally, we obtain the following mathematical model:

$$\hat{Q} = \max \left(\sum_{m \in B} e_m d_m - \sum_{l \in S} s_l p_l \right) \quad (9)$$

$$0 \leq p_l \leq p_l^{max}, \quad \forall l \in S \quad (10)$$

$$0 \leq d_m \leq d_m^{max}, \quad \forall d \in B \quad (11)$$

$$\sum_{d \in D} x_{ed} \leq \sum_{l \in S(e)} p_l, \quad \forall e \in E \quad (12)$$

$$0 \leq x_{ed}, \quad \forall e \in E, \forall d \in D \quad (13)$$

$$\sum_{e \in E} a_{ve} x_{ed} = \begin{cases} \sum_{m \in B(d)} d_m & v = s_d \\ 0 & v \neq s_d, t_d \\ -\sum_{m \in B(d)} d_m & v = t_d \end{cases}, \forall v \in V, d \in D \quad (14)$$

where:

indices:

$d = 1, 2, \dots, D$	demands for bandwidth
$v = 1, 2, \dots, V$	network nodes
$e = 1, 2, \dots, E$	network links
$l = 1, 2, \dots, S$	offers for selling
$S(e)$	offers for selling particular link e
$m = 1, 2, \dots, B$	offers for buying
$B(d)$	offers for buying particular demand d

parameters:

a_{ve}	$= 1$ if link e originates in node v , $= -1$ if e terminates in node v , $= 0$ otherwise
s_d	source node for demand d
t_d	sink node for demand d
s_l	selling price for l -th offer
e_m	buying price for m -th offer
p_l^{max}	maximal volume for l -th offer
d_m^{max}	maximal volume for m -th offer

variables:

x_{ed}	bandwidth flow serving demand d allocated to the link e
p_l	contracted bandwidth capacity for selling offer l
d_m	contracted bandwidth capacity for buying offer m

3 Aggregation and disaggregation

Paper [1] considers the auction-based pricing of network bandwidth, where the utilities of particular participants are aggregated to obtain multicom-

modity flow problem with aggregated user. Authors propose disaggregation as a set of distributed auctions each for one commodity (i.e. network path). Paper [10] assumes that the exchange may concern aggregated resources, like bulk bandwidth for aggregate flows, and virtual paths, virtual private networks, or edge capacity.

Aggregating of participants' offers is useful from the market operator's point of view. When a growing number of offers is submitted on the same link (or the same demand), the liquidity and competitiveness on such market increases, as the concentration and market power decreases. Also, when there are multiple offers submitted on given bandwidth resource, it is much more easier to approach to the competitive price of such a resource.

As the detailed realizations of particular offers are not given in the solution of BACBR model, a disaggregation process is needed, which allows us to match the accepted individual buy and sell offers. From the business point of view, disaggregation of results of BACBR model assures that every buyer knows who will be responsible for its demand realization and every seller knows to whom the bandwidth is served. It may be specially important in the case of the market operator that is not concerned with the network operations or any access switches [5]. In the process of disaggregation it is possible to consider various individual constraints and requirements, not taken into account in the aggregated model.

Let us assume results of the exemplary trading process (see Fig. 5). As we can see from Fig. 5, the demand x_1^d was realized by two link sequences: $x_1^e - x_3^e - x_5^e$ (A-B-D-C) with share equal to 0.4 and by the $x_1^e - x_4^e$ (A-B-C) with share 0.6 (see Fig. 5). Let us assume, that bandwidth allocated on the path x_1^d is equal to 10 units of bandwidth (see table 1). The bandwidth allocated to link x_1^e is equal to 10, on link x_2^e is equal to zero, for links x_3^e and x_5^e allocated bandwidth is equal to 4, and finally for link x_4^e we have 6 units of bandwidth allocated.

x_{ed}	x_1^e	x_2^e	x_3^e	x_4^e	x_5^e
x_1^d	10	0	4	6	4

Table 1: Results of the exemplary trading process

Up to now we know how particular network links

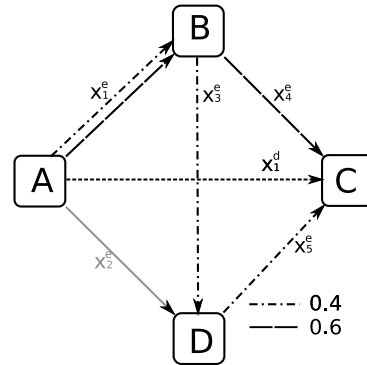


Figure 5: Solution of exemplary trading process. Realization of the path x_1^d by a bundle of links x_1^e, \dots, x_5^e

serve particular demands. However, such aggregate allocation does not give us the answer to the question: how to assign particular realization of the accepted buy offers to particular accepted sell offers. In other words, results of the exemplary offer process do not give us the answer to the question, how particular offers for buying demand x_1^d (i.e. offers d_1 and d_2) will be realized by particular sell offers (p_1, \dots, p_8).

General model for disaggregation

After solving BACBR model, we obtain the following results: the volume realization of particular sell offers $p_l \forall l \in S$, moreover we know how the sell offers are realized by the network links $e : l \in S(e)$. Respectively, we know the volume realization of particular buy offers $d_m \forall m \in B$, moreover we know how buy offers are realized by the appropriate demands $d : m \in B(d)$. Finally, we obtain the aggregated result, the realization of the demand by particular network links $x_{ed} \forall e \in E, d \in D$.

Nevertheless, we need to disaggregate x_{ed} variable, and obtain specific realization of the particular accepted buy offers d_m (which is a portion of particular demand $d : m \in B(d)$ – the offer was submitted on this demand) by particular accepted sell offers p_l (which is a portion of the particular network resource $e : l \in S(e)$ – the offer was submitted on this link). Therefore, we need to determine variable $z_{e_l d_m}$ that satisfy the following equation:

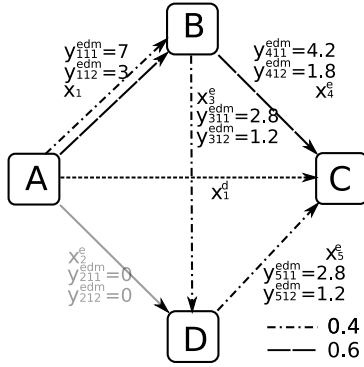


Figure 6: Exemplary solution of the first stage of disaggregation model. We can see y_{ed_m} variables which are realization of m -th buy offer (which belongs to the demand d) by link e

$$\sum_{l \in S(e)} \sum_{m \in B(d)} z_{e_l d_m} = x_{ed}, \quad \forall e \in E, d \in D \quad (15)$$

To obtain correct disaggregation, we need to find appropriate values of $z_{e_l d_m}$.

The first stage of disaggregation assumes searching for complete and coherent flows for every accepted buy offer d_m . This problem can be decomposed for every demand $d \in D$ as the particular buy offer d_m was submitted for only one demand $d : m \in B(d)$. Therefore, every accepted buy offer should flow from its source node $s_{d_m} = s_d : m \in B(d)$ to its sink node $t_{d_m} = t_d : m \in B(d)$. We obtain the following equation:

$$\sum_{e \in E} a_{ve} y_{ed_m} = \begin{cases} d_m & v = s_{d_m} \\ 0 & v \neq s_{d_m}, t_{d_m} \\ -d_m & v = t_{d_m} \end{cases}, \quad \forall v \in V, \forall m \in B \quad (16)$$

To ensure that particular flows for demand will constitute an aggregated solution, the sum of all flows y_{ed_m} has to equal x_{ed} for every link $e \in E$:

$$\sum_{m \in B(d)} y_{ed_m} = x_{ed}, \quad \forall e \in E \quad (17)$$

The variable y_{ed_m} is a result of partial disaggregation of the parameter x_{ed} . The y_{ed_m} is disaggregated only by particular buy offers. In Table 2 and

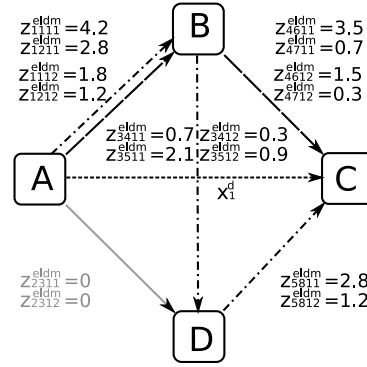


Figure 7: Exemplary solution of the second stage of disaggregation model. We can see $z_{e_l d_m}$ variables which are realization of m -th buy offer (which belongs to the demand d) by the l -th sell offer (which belongs to the link e)

on Fig. 6 we can see exemplary result of the first stage of disaggregation. We can treat the y_{ed_m} variable as realization of m -th buy offer (which belongs to the demand d) by link e .

		p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8
		6	4	0	1	3	5	1	4
x_{ed}		x_1^e	x_2^e	x_3^e	x_4^e	x_5^e			
$d_1 = 7$	x_1^d	7	0	2.8	4.2	2.8			
$d_2 = 3$	x_1^d	3	0	1.2	1.8	1.2			

Table 2: Results of first stage of disaggregation. Buy offers are correctly disaggregated

The first stage of disaggregation is a general model with some degrees of freedom, so the number of feasible solutions can be often enormous.

The second stage of disaggregation assumes disaggregation of realization of m -th buy offers for demand $d : m \in B(d)$, to the particular link e . As the result, we obtain the disaggregated variables $z_{e_l d_m}$, which correspond to the realization of particular accepted buying offer d_m by the particular accepted selling offer p_l . Note that the buying offer d_m belongs to the demand $d : m \in B(d)$, because it was submitted for this demand. Also, the selling offer p_l belongs to the network link $e : l \in S(e)$, because it was submitted for that link.

General model for the second stage of disaggregation is as follows:

$$y_{ed_m} = \sum_{l \in S(e)} z_{e_l d_m}, \quad \forall e \in E, m \in B \quad (18)$$

$$\sum_{m \in B} z_{e_l d_m} = p_l, \quad \forall l \in S \quad (19)$$

The first equation (18) is responsible for disaggregation of variables y_{ed_m} into variables $z_{e_l d_m}$. The second equation (19) is responsible for correct allocation of variables $z_{e_l d_m}$ to obtain values of accepted offers. In Table 3 and on Fig. 7 we can see exemplary results of the second stage of disaggregation. We can see, that after solving both stages, we obtain correct disaggregation, i.e. variables $z_{e_l d_m}$.

		p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8
		6	4	0	1	3	5	1	4
x_{ed}		x_1^e	x_2^e	x_3^e	x_4^e	x_5^e	x_6^e	x_7^e	x_8^e
$d_1 = 7$	x_1^d	4.2	2.8	0	0.7	2.1	3.5	0.7	2.8
$d_2 = 3$	x_1^d	1.8	1.2	0	0.3	0.9	1.5	0.3	1.4

Table 3: Exemplary results of the second stage of general model for disaggregation

As in the previous stage, this disaggregation is also general, so there are possible multiple feasible solutions.

Combined disaggregation. Two stages of disaggregation can be combined into one process. We can replace y_{ed_m} from (16) with (18), what results in:

$$\sum_{e \in E} a_{ve} \sum_{l \in S(e)} z_{e_l d_m} = \begin{cases} d_m & v = s_{d_m} \\ 0 & v \neq s_{d_m}, t_{d_m} \\ -d_m & v = t_{d_m} \end{cases} \quad (20) \\ \forall v \in V, \forall m \in B$$

Equation (20) plus (19) states general single-stage model for disaggregation. As there are many feasible solutions to the general model, many additional specific requirements can be incorporated into the process. Below we present only one simple example of disaggregation method.

Proportional disaggregation. The simplest disaggregation is the proportional method. It divides accepted buying offers proportionally, according to proportionality between selling offers:

$$z_{e_l d_m} = \frac{p_l d_m}{\sum_{l \in S(e)} p_l} x_{ed} = \frac{p_l d_m}{\sum_{m \in B(m)} d_m} \quad (21)$$

As the result of such disaggregation we obtain the following results (see Table 3). We can observe that selling offer p_1 , which is a portion of link x_1^e , serves the buying offer d_1 , which is a portion of demand x_1^d , with 4.2 units of bandwidth.

4 Summary

The proposed BACBR model for balancing aggregated communication bandwidth resources assumes aggregation of particular offers and resources, which results in more concise aggregate optimization problem for clearing the multicommodity auction. The solution to the model determines aggregated results, so the need for disaggregation process appears. We have described two phases of the general disaggregation model and showed that it may be performed in one combined disaggregation process. A simple proportional disaggregation method was also proposed. Our future line of research includes exploiting some freedom in the disaggregation methods to take into account various individual participants constraints and requirements.

Acknowledgements

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